SAP Short-term Postdoctoral Grant Report

Truthlikeness Applied: Estimating Closeness to the Truth for Scientific Laws

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I. Introduction

Truthlikeness is a property of a theory or a proposition that represents its *closeness*, *similarity* or *likeness* to the truth. As an intuitive example, consider the problem regarding the number of planets in our Solar System and the two potential answers "a) 10 planets" and "b) 20 planets". Although a) and b) are false regarding the true answer "8 planets", a) seems to *correspond better* to the facts or to be a *better description* than b), being a) *closer to the truth* than b).

As with many other philosophical concepts, one must distinguish between the *semantic* and the *epistemological* problem. The former has to do with the philosophical definition of the notion (i.e. how it is meaningful to claim that a proposition T2 is more truthlike than another proposition T1) while the latter has to do with our epistemic access to the defining conditions (i.e. how, given some evidence, it is rational to claim or estimate that a theory T2 is more truthlike than a theory T1). My research before this grant had dealt exclusively with the semantic problem in the case of scientific laws. I developed a modification of Niiniluoto's proposal (1987) that defines truthlikeness for laws as a function of two factors: *accuracy* and *nomicity*. The research gave rise to two papers (García-Lapeña, 2021, 2023), one dealing with deterministic laws and another dealing with probabilistic laws.

The SAP grant has enable me to expand the framework developed for deterministic laws (DL) to deal with the *epistemological* problem, i.e., how to estimate, given some evidence, that a law L2 is closer to the true law L* than a law L1. This has involved two steps. First, developing a satisfactory "estimated-truthlikeness" definition for DL. Second, testing the proposal applying it to a concrete case study regarding classical thermodynamics.

II. Step 1

For Step 1, I started with my definition of truthlikeness for DL, which involves a function of two factors: *accuracy* and *nomicity*. The degree of accuracy between a DL $f^X(x)$ and the true DL $f^T(x)$ can be defined by the Euclidean distance (in accordance with Niiniluoto's 1982, 1987, 2018 proposal):

$$d^{eu}(X,T) = \left(\int |f^X(x) - f^T(x)|^2 \, dx\right)^{\frac{1}{2}}$$

In García-Lapeña (2021, 2023), I argue that *accuracy* represents a necessary but not sufficient condition, and that an additional factor, *nomicity*, is needed. *Nomicity* represents the qualitative behaviours implied by a law that are not captured by value comparison and can be measured by the Euclidean distance between the corresponding derivatives:

$$d^{eu}(X',T') = \left(\int_n^m |f^{X'}(x) - f^{T'}(x)|^2 \, dx \right)^{\frac{1}{2}}$$

Then, the normalized degree of truthlikeness of a law X, $Tr(X)$, can be defined as:

$$Tr(X) = \frac{1}{\left(1 + d^{eu}(X,T)\right)} \frac{1}{\left(1 + d^{eu}(X',T')\right)}$$

Therefore, an *estimation of* $Tr(X)$ can be reduced to an estimation of the accuracy and the nomicity factors.

For accuracy, I proceed by assuming a simple case where the target system is represented according to two quantities, X and Y, and considering a set of n empirical observations $(x_1^T, y_1^T), \ldots, (x_n^T, y_n^T)$. Then, the *estimated degree of accuracy* of a law X, $E[d^{eu}(X,T)]$, can be calculated by comparing, for each empirical point, the true value y_i^T with the predicted value y_i^X (this proposal roughly matches with Niiniluoto's 1987, 2018):

$$E[d^{eu}(X,T)] = \left(\sum_1^n (y_i^T - y_i^X)^2 \right)^{\frac{1}{2}}$$

Nomicity, however, was harder to estimate, as the "true function" is unknown and the values of the derivative at the points $\left(x_i^T, y_i^T\right)$ cannot be empirically observed. The only way to proceed was to rationally estimate the value of the derivative of the true function for each empirical point. To estimate them, and after several considerations, I appealed to a method of interpolation known as *cubic splines*. Roughly, a *cubic spline interpolation* results in a piecewise function with each of its pieces being a three order polynomial. This method seemed appealing, for it instantiates the *minimum curvature property* (Holladay, 1957). This implies, roughly, that cubic splines can be understood as defining the "shortest" plus "smoothest" possible path between a set of known points. In this way, assuming a metaphysical principle of *simplicity* in nature (where simplicity is understood in the mentioned sense), cubic splines seem to provide a good estimation of the true path near the empirical points.

Now, the *estimated degree of nomicity* of a law X, $E[d^{eu}(X',T')]$, can be calculated by comparing, for each empirical point, the value of the estimated true derivative ($y_i^{T'}$) with the value of the derivative of law X ($y_i^{X'}$) (which we can calculate):

$$E[d^{eu}(X',T')] = \left(\sum_1^n \left(y_i^{T'} - y_i^{X'}\right)^2 \right)^{\frac{1}{2}}$$

III. Step 2

The next step was to test the proposal applying it to a concrete case study regarding classical thermodynamics. Here I conducted a historical analysis of the development of *gas laws* since the well-known Ideal gas law. I identified eight relevant modifications of the Ideal gas law: *Van der Waals*, *Dieterici*, *Berthelot*, *Redlich-Kwong*, *Peng-Robinson*, *Beattie-Bridgeman* and *Benedict-Webb-Rubin*.

I then selected a sample of 421 empirical observations of nitrogen in gas state, which, as raw data, were quite difficult to find. The data was taken from different sources (Figure 1) and covers a wide range of pressures and temperatures, being a good representation of nitrogen's gas region.

Summary of P-V-T data

Source	**Number of points**	**Temperature range (K)**	**Pressure range (atm)**
Canfield *et al.* (1962)	12	273	2.0-33.0
Youngglove and McCarty (1980)	222	80-330	0.2-14.8
Duchek *et al.* (1988)	30	273-313	4.9-29.6
Pieperbeck *et al.* (1991)	19	293	0.9-29.4
Nowak *et al.* (1997)	138	98-340	0.9-33.5
Overall	421	80-340	0.2-33.5

Figure 1

I then proceed to calculate the *estimated accuracy*, *estimated nomicity* and *estimated truthlikeness* of the nine historical gas laws regarding the chosen sample, using the framework developed in Step 1. Calculations were tough and required the development of a small python code. The results are given in Figure 2.

Estimated degrees of *Accuracy*, *Nomicity* and Truthlikeness regarding Nitrogen

	Accuracy		*Nomicity*		$d^{an}(X,T)$	**Truthlikeness**	**Tr*10⁴**	**Ranking**
IG	**75.6**	0.00	**285**	0.00	5,303	**0.000046**	**0.46**	**8**
VW	**14.3**	61.3	**51.1**	234	1,306	**0.001258**	**12.58**	**6**
BE	**133**	-57.9	**471**	-186	8,704	**0.000016**	**0.16**	**9**
DI	**16.7**	58.8	**63.2**	221	1,481	**0.000879**	**8.79**	**7**
RK	**4.7**	70.9	**32.5**	252	1,048	**0.005199**	**51.99**	**2**
RKS	**2.5**	73.0	**33.2**	251	1,098	**0.008235**	**82.35**	**1**
PR	**10.9**	64.7	**43.0**	242	1,198	**0.001908**	**19.08**	**5**
BB	**4.3**	71.3	**46.3**	238	1,222	**0.003980**	**39.80**	**4**
BWR	**5.2**	70.4	**35.8**	249	1,113	**0.004386**	**43.86**	**3**

Figure 2

A detailed analysis of the results goes beyond the scope of this report. In general, the results match with the standard scientific considerations about the laws and with the evolution of the historical episodes, giving plausibility to the epistemological proposal.

4. Output

The SAP grant has enable me to conduct the research mentioned in Step 1 and Step 2. After the research, I manage to write a paper presenting the proposal and the results. After polishing some final details, the paper will be sent soon to some high quality philosophy of science journal. I am deeply thankful to the Society for Applied Philosophy for considering my application worthy of the scholarship.

References

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